Teaching Vocabulary for Conceptual Understanding in the Mathematics Classroom

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What are some best practices for developing students to become knowledgeable about mathematics, think mathematically, and become mathematical communicators? The answer to this question lies in the understanding of how students learn. Often times, learning is segregated in students’ minds because they learn different aspects of the world in different disciplines. Additionally, within these disciplines they learn in a decontextualized manner leaving students with many segregated bits of knowledge and skills. Mathematics is known for its work with numbers. Yet, often times in the real world, problems do not seem to come solely in the form of numbers. In mathematics, educators often find that students are not able to apply computational skills for problem solving due to a lack of conceptual understanding. We also often find that students are not able to communicate or relay their mathematical ideas written or orally to demonstrate their level of understanding. In order for students to be able to know, do, think, and communicate mathematically, a conceptual foundation for mathematics needs to be developed. Because “[most] problems encountered by students in courses ranging from algebra through calculus and into courses on beginning proofs points to the many language-based misconceptions that students develop” (Krussel, 1998, pg. 440), this paper focuses on strategies to tackle these language-based misconceptions.

Many critique the United States educational system as a whole, which indirectly critiques the best practices utilized in the American classroom setting. Stigler & Hiebert reveal, “What we can see clearly is that American mathematics teaching is extremely limited, focused for the most part on a very narrow band of procedural skills…they spend most of their time acquiring isolated skills through repeated practice” (Stigler & Hiebert, 1999, pg. 10-11). If mathematics teaching is limited, then most likely there is a direct correlation that would reveal the limited amount of mathematical knowledge students are learning as well. If this is true, I want to question one instructional tool in mathematics that may most often be overlooked or misused: vocabulary development. What if vocabulary instruction for conceptual understanding is one of the solutions that would lead to less time spent “killing and drilling” basic procedural skills? Much research suggests it is a solution. Besides many critics general belief that the teaching within the American school model is inadequate, thus leading to the so-called “teaching gap,” there is little research that opposes the suggestion of the teaching of vocabulary. Holistically, the literature in the mathematics and literacy field reveals the overall benefits and gains for explicitly teaching mathematical vocabulary for enhancement of conceptual understanding. In fact, “Math vocabulary is inextricably bound to students’ conceptual understanding of mathematics” (Dunston & Tyminski, 2013, pg. 40). And because “many students view the language of mathematics as being a foreign language,” teaching the language of mathematics and utilizing vocabulary for conceptual understanding are vital tools for improving students’ mathematical understandings and in turn, their achievement (Dunston & Tyminski, 2013, pg. 40).

Rubenstein (2007) states, The “*Principles and Standards for School Mathematics* reminds us that communication is central to a broad range of goals in goals of mathematics education…[and] one part of communication is acquiring mathematical language and using it fluently” (pg. 200). If communicating mathematical terminology is necessary for overall mathematics achievement and aligns with NCTM’s principles and standards for mathematics, it is clear that there is a demand for students to possess a rich understanding of mathematical vocabulary. Because “vocabulary knowledge is acquired through a combination of incidental learning (encounters with unfamiliar words through reading and listening) and direct instruction (the teaching of specific word meanings),” educators need to be sure that direct instruction of vocabulary is aimed to diminish students’ misunderstandings they develop through incidental learning (Dunston & Tyminski, 2013, pg. 39). This paper aims to decode “what aspects of teaching and learning [mathematical] language [through vocabulary] can help develop excellent mathematics teaching and can encourage students to learn in a meaningful, insightful, and productive way [that enhances conceptual understanding of mathematics]” (Krussel, 1998, pg. 436)?

Rubenstein & Thompson (2002) suggest, “Because one of the few places students have to “talk mathematics” is in our classrooms, we as educators must give attention to mathematical language learning” (pg. 111). It is well documented throughout the literature that the usage mathematical language through discourse enhances conceptual understanding. Yet Gay (2008) exposes, “Students need to know the meaning of mathematics vocabulary words—whether written or spoken—in order to understand and communicate mathematical ideas” (pg. 218). Therefore, in order to get students to a level where they can communicate their mathematical ideas effectively, it is a teacher’s responsibility to guide them through understanding the language of mathematics. In particular, “Mathematics vocabulary instruction is particularly important in the middles grades because this is when ‘the serious of development of the language of mathematics begins’ and focused on numbers multiplicative structures and relationships” (Dunston & Tyminski, 2013, pg. 40). If mathematics is to be communicated appropriately, both written and orally, students need an exceptionally deep conceptual understanding of mathematical vocabulary in the middle grades for a solid foundation in the higher, more accelerating math classes at the secondary level.

Some educators, like Cass (2009) come across discrepancies with vocabulary in their classrooms: “The students were sometimes able to spout definitions, but they seemed to have little conceptual understanding” (pg. 148). Because his students’ lack of conceptual understanding was causing mathematical discourse problems in the classroom, he “began to read the discourse literature [and] through these readings, [he] learned more about *mathematical vocabulary* and vague referencing” (Cass, 2009, pg. 148; emphasis added). As educators recognize students’ lack of understanding through classroom discourse, the literature reveals the importance of understanding mathematical terminology for effective mathematics talk. In addition, Cass notes the necessity for clear, precise mathematical communication. When he noticed his students’ inappropriate usage of vocabulary terminology, he began “to support students’ use of mathematical vocabulary in whole-class discussions [by] revoicing in more careful ways…in mathematically precise ways” (Cass, pg. 155). Herbal-Eisenmann (2009) reveal “Revoicing [is] the reuttering of another person’s speech through repetition, expansion, rephrasing, and reporting… revoicing serves many purposes, [one] of which [is] modeling correct use of mathematical language” (pg. 33-34). For this strategy to be effective for embedding a deeper understanding of mathematical concepts and improving the vocabulary usage for students, “Teachers must have a mastery of [mathematical] vocabulary and use words correctly as they teach” (Gay, 2008, pg. 218). But as I mention previously, before mathematical discourse can improve, more effective direct vocabulary instruction for conceptual understanding must take place in the classroom.

Many math educators may question why it is their responsibility to teach and discuss vocabulary explicitly in the math classroom—that is the language arts job, right? Researchers (Gay, 2008; Rubenstein & Thompson, 2002; Rubenstein, 2007; Thompson & Rubenstein, 2000) reveal numerous categories of difficulty for students’ misunderstandings of mathematical vocabulary, thus hindering their overall mathematical communication, learning, and achievement which is why is direct instruction of vocabulary is deemed necessary:

1. *Some words are shared by mathematics and everyday English and have comparable meanings, but they have a more distinct meaning in mathematics.* When students hear these words used across outside of the context of mathematics and then inside the mathematics classroom, they often struggle to decipher how the terms in their everyday language is distinctly different from the mathematical definition. A few examples of terms that fit this category are: even, odd, power, right, similar, event, factor, prime, expression, base, radical, combination, function, mode, limit, slope, difference, reflection, variable, and foot.
2. *Some mathematical terms are homonyms or homophones with everyday English words.* Aligning with the first category of difficulty, words like some, pie and compliment are possible every day usage vocabulary words for students. When introduced to these words in a mathematical context, they may misunderstand or misuse the word based on a student’s prior knowledge. Explicitly addressing these misconceptions may help enhance students’ conceptual understanding.
3. *Some words have more than one mathematical meaning when used as a different part of speech.* For instance when defining a four sided figure with all congruent sides and four right angles, it is defined as a square. Yet, when one squares a number, he/she multiplies the number by itself to get a perfect square. Also these two concepts can be bridged pictorially and conceptually, effective vocabulary instruction on usage of the term in context may clarify misconceptions for students. Other examples are: round, range, base, second, and side.
4. *Some words are found only in mathematical contexts.* Some of these terms include: quotient, denominator, isosceles, polynomial, hypotenuse, asymptote, hyperbola, algorithm, integer, parallelogram, and quadrilateral.
5. *Some mathematical concepts are verbalized in more than one way.* One example of this is the fraction one-fourth. When talking in terms of money or football, one-fourth may be communicated as a quarter. Although “as adults, we are comfortable with the varied meanings and implied understandings in words and phrases” (Rubenstein & Thompson, 2002, pg. 107), students are not. For conceptual purposes, we need to address these vocabulary issues for students to be able to have a deeper understanding and be able to better communicate mathematically.
6. *Some words are learned in pairs that often confuse students.* A few examples of these include: multiple and factor, radius and diameter, area and perimeter, numerator and denominator, and hundreds and hundredths. Although often words taught together are related mathematically, focusing on two concepts together without clear definitions distinguishing differences, students may mix up the two words and lack the conceptual understanding to apply or communicate their understandings.
7. *Some words are shared with other disciplines have different technical meanings in the two contexts.* Some words in mathematics are also explicitly taught in science, English/Language Arts, or Social Studies. If educators do not clarify a term’s specific mathematical definition, students may assume the term is the same across content areas, which would result in a misunderstanding. A few examples to fit this category of difficulty are: prism, median, power, degree, divide, and variable.
8. *A single English word may translate into Spanish or another language in two different ways.* For English Language Learners (ELLs) in a general education classroom, these types of terms may be difficult bridge conceptually. An example would be the word table. In Spanish it can be translated to “mesa” but this would mean “the dinner table.” The appropriate term for a mathematics table translates to “tabla” in Spanish. Upon hearing the word, and ELL may struggle to understand the concept and purpose of a table in mathematics.
9. *Modifiers change meanings of words in critical ways.* A few examples of these include: bisector versus perpendicular bisector, equation versus linear, quadratic, and cubic equation, and trapezoid versus right trapezoid. These modifiers (adjectives) describing the noun may seem clear cut, but for some students, putting the terms together may cause some discrepancies.
10. *Students may adopt an informal terms as if it is mathematical.* This would cause a divergence in mathematical discourse, which would cause for an excellent classroom discussion, but with being brought to attention, students may misalign mathematical terms with similar terms. Two examples of this would include diamond versus a rhombus and a corner versus a vertex.

Being knowledgeable about how these particular challenges for students’ comprehension in understanding vocabulary terminology may be a teacher’s first step in enhancing students’ overall math learning and achievement in the classroom. In addition to understanding these difficulties, a teacher must research best practices to help students overcome these challenges. Throughout the literature, many researchers propose how to bridge these conceptual gaps through the explicit teaching of mathematic vocabulary.

Because many students possess prior knowledge of some mathematical terms, one strategy suggested throughout much of the literature is connecting a mathematical term to prior knowledge. Rubenstein (2007) argues, “build[ing] connections between known and new ideas and invites higher-level thinking” (pg. 215). Students come into our classrooms with prior knowledge, and educators can utilize this prior knowledge by building bridges between what the students already know to what they should know mathematically. Not to mention, “[it is especially important] to build bridges between everyday language and mathematics for students with disabilities” because students with disabilities may struggle to understand concepts (Lee & Herner-Patnode, 2007, pg. 124). This strategy is vital for them to be able to better comprehend how their new learning can be build off something they are already familiar with. Rubenstein & Thompson (2002) contend, “A major premise of all [vocabulary] strategies is to connect new terms or phrases to ideas children already know. Children should first do activities that build concept, then express their understanding informally, and finally, when ideas solidify; learn the formal language” (pg. 108). The formal language comes after the conceptual understanding is development. This understanding is pivotal for effective usage of this strategy because educators can lead students to many misconceptions about mathematical concepts from trying to bridge a concept simply from just connecting vocabulary words.

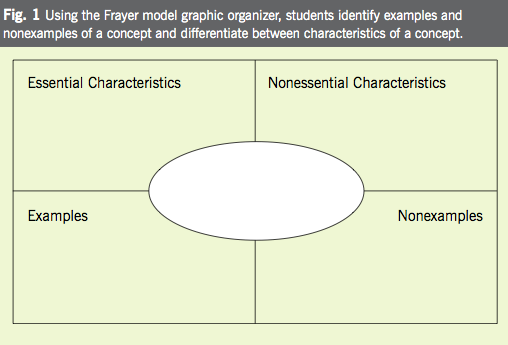
One the contrary, some teacher educators caution the usage of trying to activate students’ prior knowledge about a mathematical term to a new concept. Dunston (2013) serves as a literacy educator at Clemson University. She notes how this strategy may cause students to develop misunderstandings about a mathematical concept: One of her pre-service teachers stated, “A slice of pizza or apple pie is an example of an acute angle. Several students concluded that acute angles have at least one rounded or curved edge” (Duston & Tyminski, 2013, pg. 39). Although many researchers suggest connecting new mathematical vocabulary terms to students’ prior knowledge, educators have to approach this strategy with a holistic understanding of the vocabulary terminology to best connect it to the appropriate mathematical concept for students.

Another strategy mentioned across the literature is having students invent their own language for the mathematical concepts they are learning. Kussel (1998) reveals, “It is not enough just to be aware of the need for mathematical definitions; students need ways to develop their knowledge of mathematical language” (Krussel, 1998, pg. 439). Having students develop words to define their conceptual understanding can serve many purposes in the classroom. It may serve as a formative assessment in how well the students understand the concept. In addition, “invented language can enhance understanding” (Rubenstein, 2007, pg. 215) which is the goal of all mathematic instruction, and is, additionally, the goal of having students invent their own language to definer conceptual understanding. This strategy may also serve as one source of engagement for students to engage in rich mathematical concepts as well as helping them understand what learning truly is:

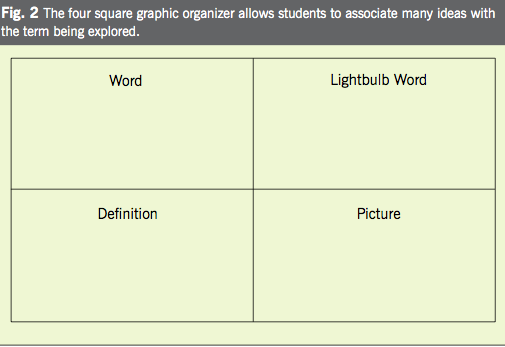
Simply withhold the formal terminology. Let students use materials to explore ideas, suggest their own terms, and explain their rationales. Of course formal terms must be introduced eventually, and students must be able to translate between informal and standard words, but by inventing they realize that terms come from people *thinking* about new ideas. This realization aligns with an important goal we have for students: to be thinkers and creators in the world of the future. (Rubenstein & Thompson, 2002, pg. 109).

More often than not, if a conceptual foundation is formed, students’ informal created terms and the formal term will be somewhat related and that connection can be easily translated to the new formal terminology. Teaching students to think and be creative is often shunned with many educators focus on standardized testing, but this strategy gets at the heart of learning. Moreover, when students are creating the terms from their understanding, they are taking ownership of their learning which is another benefit from this strategy (Rubenstein & Thompson, 2002)

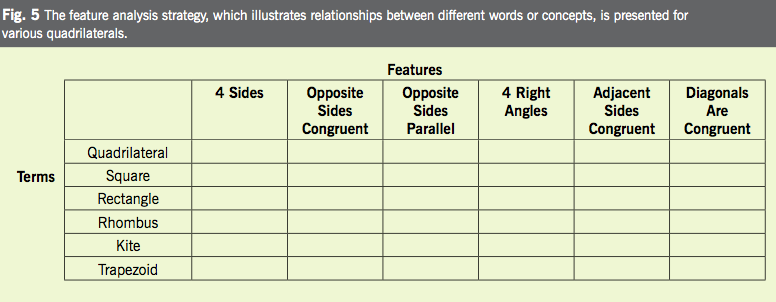
Another best practice to help utilize vocabulary for conceptual understanding is the usage of graphic organizers. Graphic organizers “help students visualize, affiliate properties to vocabulary terms, compare and contrast concepts, and distinguish hierarchy (Rubenstein, 2007, pg. 216; Dunston & Tyminski 2013; Thompson & Rubenstein 2002, 2000; Rubenstein 2007; Gay 2008). Graphic organizers can be modified for differentiation and can be adapted to meet the needs of an educator’s particular set of students. Each one serves a different purpose. One graphic organizers that was noted across the literature was the Frayer Model which, “helps students organize their thinking about a concept in the same three ways that teachers employ to teach the meaning of a concept—one the connotative use of term; two the denotative manner, and three the implicative use of the term” (Gay, 2008, pg. 220). In the Frayer model, “students identify examples and nonexamples of a concept, differentiate between characteristics that define (or are associated with) the concept, and [define] characteristics that are interesting but not important” (Dunston & Tyminski, 2013, pg. 41). Through a literacy approach, this specific graphic organizer is aimed to enhance vocabulary for overall comprehension. This strategy is not specific to a particular content area and can help support students’ vocabulary development. The example below is provided by Dunston & Tyminski (2013).



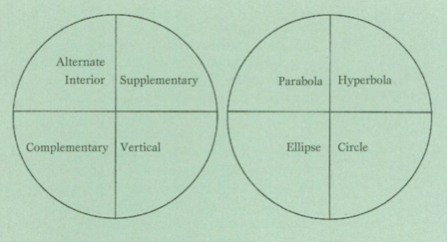
Another graphic organizer designed to enhance the conceptual understanding of a vocabulary term is the Four Square. As Dunston & Tyminski (2013) note, “[This graphic organizer] is similar to, but not as complex as, the Frayer Model” (pg. 42). The Four Square includes the term, a student’s own interpretation of the term’s definition, a student’s own term that is classified as the “light bulb word,” and a picture/figure for visual representation. The example below is provided by Dunston & Tyminski (2013).

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Two graphic organizers that aim to compare and contrast multiple concepts within mathematics are the Feature Analysis and Concept Circles. Feature Analysis “helps middle grades students conceptualize and make sense of properties of [a concept] and relationships between [multiple] concepts” (Dunston & Tyminski, 2013, pg. 43) while Concept Circles “encourages students to study words critically, relating them conceptually to one another” (Gay, 2008, pg. 221). The Feature Analysis compares different aspects within a category and can show hierarchy of terms. Below is an example provided by Dunston & Tyminski (2013) comparing different quadrilaterals.



The Concept Circle can be adapted many different ways to challenge students, The examples below provided by Gay (2008) do not reveal the overall concept the vocabulary terms are affiliated with which can challenge students to think critically to determine the overarching concept.



These two graphic organizers are meant to help students compare concepts after the individual concepts and understanding of vocabulary terms have been develop. If implemented, the two graphic organizers should come after using models such as the Frayer Model or Four Square.

In contrary to the development of mathematical concepts through graphic organizers, there are many best practices mentioned throughout the literature that aim to help students think creatively about their understanding of concepts in terms of vocabulary—analogies, metaphors, and mnemonic devices (Gay 2008; Rubenstein & Thompson 2002; Krussel 1998).

Gay (2008) notes, “Research has shown that analogies can facilitate conceptual understanding and make concepts easier to remember [because] analogies can build connections between known and new ideas” (pg. 222). Krussel also argues, “Metaphor[s are] as central to the expression of mathematical meaning, as it is to the expression of meaning in everyday language” (Krussel, 1998, pg. 439). Since mnemonic devices are typically phrases that have some sort of rhyming, this strategy may help students who are auditorial learners. Using these strategies, students are able to creatively apply their understandings of mathematical concepts to things in their everyday life and language, which allows for students to make personal connections to the concepts, thus providing students with a greater opportunity to embed the concept into their long-term memory. These strategies can help students separate concepts that are usually taught together, but have distinct differences.

Many of the categories of difficulties with vocabulary development in the mathematics classroom can be diminished from the teaching of word origins and roots (Rubenstein, 2007; Thompson & Rubenstein 2007; Rubenstein & Thompson 2002). Although this is often perceived as a English/ Language Arts teaching tactic, utilizing this literacy strategy in the math classroom can enhance students’ overall understanding of words, and thus mathematical vocabulary terms enhancing their mathematical understnadings.A multitude of literature affirms the benefits of decoding words with students and helping them understand word origins and roots: “The origins of words are often helpful in bringing language to life, making terms more meaningful, and revealing connections with related ideas” (Rubenstein, 2007, pg. 216; Thompson & Rubenstein, 2007, pg. 572-573; Rubenstein & Thompson, 2002, pg. 111). Since many common roots are shared with other disciplines and everyday English words, students can make connections and understand the overall meaning of terms at a deeper level. By “recogniz[ing] that words have etymologies and relationships to others words that provide some reason for their having been invented or selected to represent certain mathematical ideas or entities” (Rubenstein, 2007, pg. 203) teaching word origins can help diminish misunderstandings with challenging mathematical vocabulary terms.

A great deal of literature reveals the importance of integrating writing into all content areas, especially in mathematics (Rubenstein & Thompson 2002; Thompson & Rubenstein 2000; Lee & Herner-Patnod 2007; Krussel 1998). This is additionally represented through the new literacy Common Core State Standards. In order to communicate mathematical ideas through discourse using appropriate mathematical vocabulary, educators must also allow students express themselves through writing to also enhance their speaking. Although a student may be proficient in communication of mathematical ideas both written and orally, students may struggle with connecting the appropriate mathematical symbols affiliated with the vocabulary. Some best practice that utilize the bridging of concepts, terms, and corresponding symbols corresponding symbols are strategies such as mathematical graffiti (also known as word art) and picture dictionaries (Rubenstein & Thompson, 2002, pg. 109; Thompson & Rubenstein, 2000, pg. 571) An example of mathematical graffiti provided by Thompson & Rubenstein (2000) is shown below.



Mathematical graffiti can help visual learners apply their conceptual understanding to the vocabulary terms. These images can then be included in students “picture dictionaries in which are connected with written descriptions in the students’ own words (Thompson & Rubenstein, 2000, pg. 572).

As this paper argues, the “fluent use of terminology is a necessary, albeit not sufficient, condition for overall mathematics [understanding and] achievement” (Thompson & Rubenstein, 2007, pg. 568; Cass, 2009, pg. 152). In agreement with Krussel (1998) “Mathematical illiteracy continues to be far too acceptable in our society” (pg. 438). Educators need to dive into the literature and discover what best practices meet the needs of students. “Recognizing that students think and learn in many ways, we propose a spectrum of approaches, including oral, written, visual, and kinesthetic modes” that help meet the needs of each student that enters in our classroom (Thompson & Rubenstein, 2000, pg. 570). The American classrooms have been and are becoming more radically diverse in terms of demographics, cultures, and learning styles. To meet the needs of each diverse learner, we have to vary in the ways we teach mathematic vocabulary to enhance students’ conceptual understandings. As Lee & Herner-Patnod (2007) warn us, “building conceptual knowledge requires time and multiple experiences in various contexts—visual, auditory, kinesthetic” for students with disabilities (pg. 125). And “mathematical vocabulary helps students acquire the conceptual knowledge they need to understand age-appropriate concepts” (Lee & Herner-Patnode, 2007, pg. 122). Although “vocabulary should not be stressed to the degree that students feel learning vocabulary is the same as learning mathematics [because] teaching vocabulary is not teaching mathematics, it is one of the skills that must be taught” (Lee & Herner-Patnode, 2007, pg. 126).

References

Cass, T. (2009). “It,” “that,” and “what”? vagueness and the development of mathematics vocabulary. In B. Herbal-Eisenmann & M. Cirillo (Eds.), *Promoting Purposeful Discourse* (pp. 147-164). Reston, Virginia: National Council of Teachers of Mathematics.

Dunston, P., & Tyminski, A. (2013). What's the big deal about vocabulary?. *Mathematics Teaching in the Middle School*, *19*(1), 38-45.

Gay, S. (2008). Helping teachers connect vocabulary to conceptual understanding. *The Mathematics Teacher*, *102*(3), 218-223.

Herbal-Eisenmann, B. (2009). Some essential ideas about classroom discourse. In B. Herbal-Eisenmann & M. Cirillo (Eds.), *Promoting Purposeful Discourse* (pp. 29-42). Reston, Virginia: National Council of Teachers of Mathematics.

Krussel, L. (1998). Teaching the language of mathematics. *The Mathematics Teacher*, *91*(5), 436-441.

Lee, H., & Herner-Patnode, L. (2007). Teaching mathematics vocabulary to diverse groups. *Intervention in School and Clinic*, *43*(2), 121-128.

Rubenstein, R. (1996). Strategies to support the learning of the language of mathematics. In P. Elliott & M. Kenney (Eds.), *Communication in Mathematics, K-12 and Beyond* (pp. 214-218). Reston, Virginia: National Council of Teachers of Mathematics.

Rubenstein, R. (2007). Focused strategies for middle-grades mathematics vocabulary development. *Mathematics Teaching in the Middle School*, *13*(4), 200-207.

Rubenstein, R., & Thompson, D. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics* , *9*(2), 107-112.

Stigler, J., & Hiebert, J. (1999). *The teaching gap, best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.

Thompson, D., & Rubenstein, R. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *The Mathematics Teachers*, *93*(7), 568-574.